**Tasks**

# **Task 1. Ordinary differential equations.**

To solve the differential equation



with initial conditions



Table of parameters

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **variant** | ***t*0** | ***a*** | ***b*** | ***c*** |
| 1 | 0 | 4 | 2 | -1 |
| 2 | 1 | -4 | 1 | -2 |
| 3 | -1 | 1/4 | -1 | 2 |
| 4 | 1 | 9 | 3 | 0 |
| 5 | 2 | 1 | 2 | 2 |
| 6 | 0 | -1/4  | 0 | -1 |
| 7 | -1 | -9 | -1 | -2 |
| 8 | -2 | -1 | 1 | 4 |
| 9 | 0 | 1/9 | -1 | 0 |
| 10 | 1 | -1/9 | 1 | -1 |

**Task**.

1. Find the general solution of the given Cauchy problem. This is  if *a=λ*2 and  if *a=-λ*2.
2. Using the initial conditions, find the constant *c*1 and *c*2.
3. Put these constant to the formula of the general solution.
4. Make sure that the result satisfies, in reality the given equations and initial conditions.

# **Task 2. First order partial differential equations**

Consider the first order partial differential equation

 , 0<*x*<*L*, 0<*y*<*M*, (1)

where *a*, *L*, *M* are given constants. If *a*>0, then we can have the boundary conditions

  (2)

or

  (3)

If *a*<0, then we can have the boundary conditions

  (4)

or

 , (5)

where *ϕ* and *ψ* are given functions.

It is necessary to find the solution of the given problem, using characteristic method. Check that the result is, in reality, the solution of the problem.

Table of parameters

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **variant** | ***a*** | **boundary****conditions** | ***L*** | ***M*** | *ϕ*(*y*) | *ψ*(*x*) |
| 1 | 2 | (2) | 1 | 2 | sin π*y* | sin 2π*x* |
| 2 | -2 | (4) | 2 | 1 | cos 2π*y* | cos π*x* |
| 3 | 1/2 | (3) | π/2 | π | -sin *y* | -sin 2*x* |
| 4 | -1/2 | (5) | π | π/2 | -cos 2*y* | -cos *x* |
| 5 | 3 | (2) | π | π/2 | sin 2*x* | sin *y* |
| 6 | -3 | (4) | π/2 | π | cos *y* | cos 2*x* |
| 7 | 1/3 | (3) | 2 | 1 | -sin 2π*y* | -sin π*x* |
| 8 | -1/3 | (5) | 1 | 2 | -cos π*y* | -cos 2π*x* |

# **Task 3. Classification of partial differential equations and its canonic forms**

Determine the sets, where the given equation has the concrete type. Transform it to the canonic form for any considered type.

**Table of equations**

|  |  |
| --- | --- |
| variant | equation |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |

It is necessary perform the following steps:

1. Calculate the value of the discriminant *D.*
2. Using the sign of *D* determine the sets of the plane *xy*, where the equation has the concrete type.
3. For the hyperbolic case, write two characteristic equations.
4. Find its general solutions.
5. Write these general solutions in the form  and .
6. Determine the new variables  
7. Calculate the coefficients of the equation in the new variables by the given formulas.
8. Determine the canonic form of the given equation for the hyperbolic case.
9. For parabolic case, consider the unique characteristic equation, determine variable  by previous method with arbitrary variable *η*, and repeat the actions of hyperbolic case.
10. For elliptic case, consider the first characteristic equation with complex parameters, find its general solution, write it is the form , choose the functions *ξ* and *η* as new variables, and repeat the actions of hyperbolic case.

# **Task 4. Running waves**

Consider theCauchy problem for the vibrating string equation

*utt = uxx* , -∞ < *x* < ∞, *t* > 0;

*u*(*x*,0) = *ϕ*(*x*), *ut*(*x*,0) = 0.

The function *ϕ* is given, see the following images:



 Variant 1 Variant 2 Variant 3



 Variant 4 Variant 5 Variant 6



 Variant 7 Variant 8

It is necessary to use the D'Alembert formula and analyze the corresponding running waves. Show all steps of the phenomenon.

# **Task 5. Oscillation of the string with fixed ends**

Consider first order boundary problem for the vibrating string equation:

*utt = a2 uxx*, 0 < *x* < *L*, *t* > 0,

*u*(0,*t*) = 0, *u*(*L*,*t*) = 0, *t* > 0,

*u*(*x*,0) = *ϕ*(*x*), *ut*(*x*,0) = *ψ*(*x*), 0 < *x* < *L*.

Table of parameters

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| variant | *L* | *a* | *ϕ*(*x*) | *ψ*(*x*) |
| 1 | 1 | 2 | - sin π*x* | 0 |
| 2 | π | 1 | 0 | sin *x* |
| 3 | 2 | ½ | sin 2π*x* | 0 |
| 4 | 2π | 2 | sin (*x/*2) | 0 |
| 5 | 1 | 1 | 0 | sin π*x* |
| 6 | π | ½ | sin *x* | 0 |
| 7 | 2 | 2 | 0 | sin 2π*x* |
| 8 | 2π | ½ | 0 | - sin (*x/*2) |

It is necessary to find the solution of the problem, to show the graph (position of the string for the different time points), and to give the physical interpretation of the results.

# **Task 6. Oscillation of the string with free ends**

Consider second order boundary problem for the vibrating string equation:

*utt = a2 uxx*, 0 < *x* < *L*, *t* > 0,

*ux*(0,*t*) = 0, *ux*(*π*,*t*) = 0, *t* > 0.

*u*(*x*,0) = *ϕ*(*x*), *ut*(*x*,0) = *ψ*(*x*), 0 < *x* < *L*.

Table of parameters

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| variant | *L* | *a* | *ϕ*(*x*) | *ψ*(*x*) |
| 1 | π | 1 | 0 | sin *x* |
| 2 | 1 | 2 | - cos π*x* | 0 |
| 3 | 2π | 2 | cos (*x/*2) | 0 |
| 4 | 2 | ½ | cos 2π*x* | 0 |
| 5 | π | ½ | cos *x* | 0 |
| 6 | 1 | 1 | 0 | cos π*x* |
| 7 | 2π | ½ | 0 | - cos (*x/*2) |
| 8 | 2 | 2 | 0 | cos 2π*x* |

Task:

1. Find the solution of the problem.
2. Check that this is, in reality the solution.
3. Show the graph (position of the string for the different time points).
4. Give the physical interpretation of the results.
5. Forced vibrations of the string.
6. Heat equation with known temperature at the boundary.
7. Heat equation with known heat flux through the boundary.
8. Heat equation in the presence of heat sources.
9. Variational method in mathematical physics problems.
10. Laplace equation in a circle.
11. Green function method mathematical physics problems.
12. Finite difference method for mathematical physics problems.
13. Inverse problems of mathematical physics.

# **Task 7. Oscillation of the string under exterior force**

Consider the movement of the under string exterior force characterized by the given function *f.* This is described by non-homogeneous vibrating string equation

*utt = a2 uxx* + *f*(*x*,*t*), 0 < *x* < *L*, *t* > 0.

Suppose the string is in the state of equilibrium and has zero velocity at the initial time. Then we have the initial conditions

*u*(*x*,0) = 0, *ut*(*x*,0) = 0, 0 < *x* < *L*.

The ends of the string can be free or fixed, i.e. we have one of the following boundary conditions

 *u*(0,*t*) = 0, *u*(*π*,*t*) = 0, *t* > 0; (\*)

 *ux*(0,*t*) = 0, *ux*(*π*,*t*) = 0, *t* > 0. (\*\*)

Table of parameters

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| variant | boundary condition | *L* | *a* | *f* |
| 1 | \* | 1 | 2 |  –sin π*x* |
| 2 | \*\* | π | 1 | cos *x* |
| 3 | \* | π | ½ | –2 sin *x* |
| 4 | \* | 1 | 3 | sin 2π*x* |
| 5 | \*\* | π | ½ | cos 2*x* |
| 6 | \*\* | 1 | 1 | –cos π*x* |
| 7 | \*\* | 1 | 2 | cos 2π*x* |
| 8 | \* | π | ½ | sin 2*x* |

Task:

1. Determine the solution of the problem as sinus Fourier series for the boundary conditions (\*) and cosine Fourier series for the boundary conditions (\*\*).
2. Find the Fourier coefficient of the parameters of the system.
3. Solves ordinary differential equations with respect to the Fourier coefficient of the solution of the problem.
4. Check that this is, in reality the solution of the boundary problem.
5. Show the graph (position of the string for the different time points).
6. Give the physical interpretation of the results.